ONTARIO. MATHEMATICS. INTERMEDIATE DIV. 1978

Ontario

Ministry of Education

Mowat Block
Queen's Park
Toronto, Ontario
M7A 1L2

GRADE 9 GENERAL, NUMERICAL METHODS

NOTES FOR TEACHERS

CONTENTS

N1	Number Applications	5 pages
N2	Statistics	15 "
N3	Powers, Roots	17 "

(Notes for N4 to N7 are under preparation

and will be distributed later)

The resource notes in this module are related to the Grade 9 General, Numerical Methods Strand for Intermediate Division Mathematics 1977, Draft Copy. They are intended for use by teachers and board curriculum committees as they plan the mathematics program for their schools.

Ministry of Education Ontario

GRADE 9 GENERAL NUMERICAL METHODS

SECTION 1: NUMBER APPLICATIONS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 7, 11
	Ed PJ Div:	Pages 61-74 From Counting to Calculation (resource document)
	Gr 7:	N 1; N 2; N 4de; N 7a; N 8bd-i
	Gr 8:	N 1; N 2cd; N 3b-e; N 6; N 7; A 1b; A 3; G 3ab
PRESENT	Gr 9 Gen:	N 2; N 3; N 4; N 5; N 6; N 7c; A 1; A 5; G 1d; G 2e
	Gr 9 Adv:	N 1; N 2; N 3; N 4; N 5; N 6; A 1; A 5; G 1b; G 1d; G 2d; G 4c
FUTURE	Gr 10 Gen:	N 1; N 2; N 3; N 4; N 5; N 6; N 7; N 8; G 1b; G 3bcd
	Gr 10 Adv:	N 2; N 3; N 4; A 1d; A 3; G 1a-e; G 2b; G 5bcdf

a) Practising computational skills with whole numbers, decimals, and integers using games, puzzles, and activities; applications in real-life situations

Many of the students in this program will require additional experiences both in learning and understanding basic number skills and in reinforcing these skills through a variety of activities. Because the students are at various stages in their skill development, routine drill activities for the entire class may be boring for some students but too difficult for others. Drill exercises should be suited to the individual and assigned throughout the year as a consolidation of skills learned in response to specific needs of both individuals and groups.

This section indicates that games, puzzles, activities, and applications should be an integral part of the program. These should be used to motivate the students and to maintain their interest throughout the year. Through their use, individual needs may be identified and remedied on an on-going basis. Once this approach is established, many of the students may become so interested that the games, puzzles, and activities will spill over into out-of-class time. It is not intended that this section should come first in the school year, nor should it be taught as a complete unit at one time.

In modern society, game-playing has suffered a decline in many homes. It may, therefore, be a new experience for some students and will require careful introduction. Ideally there should be sufficient copies of a game so that, at the appropriate time, all the students can be engaged in the same activity. Because of the expense, however, it may be necessary to use several different games at the same time. In this case, a useful strategy is to select a few students, say five per class, and spend time after school teaching them how to play the various games. In a games period every group will, therefore, have a leader, freeing the teacher to circulate or to participate in the various games.

Many commercial games are suitable for the mathematics classroom, and often their use can be identified directly from their names. Some of them are:

Ranko

Vector

3-D Tic Tac Toe

Avalanche

Equations

Tuff

Some games can be extended by the mathematics teacher. For example, Ranko can be expanded by additional sets of cards to include integers, fractions, and decimals.

The Ministry of Education would be pleased to hear from teachers who have integrated game-playing into their programs, with the name of the game, its mathematical importance, and grade suitability.

Applications in real-life situations

'Real life' must be interpreted in terms of the student's experience and interest. School itself is a part of 'real life' to students, and the solution of any problem posed in a mathematics class is a real-life problem in that sense. However, teachers should strive to use examples that relate to the world outside the classroom.

In this grade there is some merit in drawing the student's attention to the world of the consumer and the analysis of advertisements.

Example:

A certain ball-point pen manufacturer produces a refill. On the package is the claim:

Ink Content and Performance Comparison

"Writes longer than ordinary ball-points. Laboratory data on request."

Upon request, the following data is obtained:

	Volume (cm^3)	Writing distance (m)
Narckur Economy	1.02	8365
Product 'PMJ'	0.77	6189
Product 'S'	o.89	4732
Product 'C'	0.45	4073
Product 'PMP'	0.31	2346
Product 'B'	0.35	1218

Obviously valid comparisons would be by metres per ¢, or by metres per cm^3. Using the latter basis we find that

product 'C' is the most efficient; then three pens have about the same efficiency; the other two pens are much less efficient.

Ministry of Education Ontario

GRADE 9 GENERAL NUMERICAL METHODS

SECTION 2: STATISTICS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 6, 11
	Ed PJ Div:	Pages 61-68, 72-74
		Maps and Graphs (resource document)
	Gr 7:	N 1; N 2; N 3; N 4ab; A 1; A 3b
	Gr 8:	N 1; N 2; N 3; N 5; N 6; A 2b; A 3; A 4
PRESENT	Gr 9 Gen:	N 1; N 4; N 5; N 6c; N 7; A 1; A 4; A 5
	Gr 9 Adv:	N 1; N 2; N 4; N 5; N 6; A 1; A 4abc; A 5
FUTURE	Gr 10 Gen:	N 1; N 2; N 3; N 5; N 6; N 8; A 1ab; A 2a
	Gr 10 Adv:	N 3; N 4; A 1a; A 3; A 4

a) Data gathering methods

The following methods for gathering data have both advantages and disadvantages.

. Questionnaires are relatively easy to analyse; however, the responses are not always reliable since all people do not interpret questions in the same way, or are not completely truthful in their responses.

. Telephone interviews are more expensive and time consuming than questionnaires; however, it is usually possible to obtain reliable answers by explaining the questions in detail.

. Personal interviews require tact and experience if they are to be done well without offending anyone.

Students should appreciate that the data obtained by a survey will vary with the sampling procedure.
For example, a telephone survey excludes people who do not have a telephone, and thus does not necessarily represent the opinions of the entire population.

The questions used in a survey must be constructed with great care in order to avoid loaded and forced-choice questions. For example, a question such as:

"Do you prefer Smile toothpaste or Yuck toothpaste?"

is both loaded and forced-choice. The word 'Smile' has more pleasant associations than the word 'Yuck', and in some cases the interviewee may actually prefer a toothpaste

other than 'Smile' or 'Yuck'.

Measurement activities should be included in this section; for example, the length distribution of all the pencils in the classroom, or the time taken to solve a straight-forward problem using paper and pencil or a calculator.

b) Organizing data into frequency tables

In Grade 9, the data that is to be collected should not require the formation of class intervals when it is organized. In fact the first experiences might well be related to non-numerical data, as in the following example.

The distribution of eye colour

Colour	Tally	Frequency
Blue	~~IIII~~ ~~IIII~~ I	11
Brown	~~IIII~~ ~~IIII~~ III	13
Green	II	2
Hazel	III	3

It is also possible to examine situations in which numbers are used but in which class intervals are not needed, as in the following example.

Distribution of ages in a house team

Age	Tally	Frequency
14	III	3
15	~~IIII~~ III	8
16	~~IIII~~ I	6
17	III	3
18	I	1

c) Representing data in graphical form

Data representations could include bar graphs, histograms, circle graphs, line graphs, and pictographs.

Bar Graphs

The bar graph has been introduced in earlier grades and is characterized by data that is not functionally related (the range does not depend upon the domain in any predictable way; there is no natural order for the data).

Visually, bar graphs are characterized by either vertical or horizontal bars, usually separated from each other by spaces.

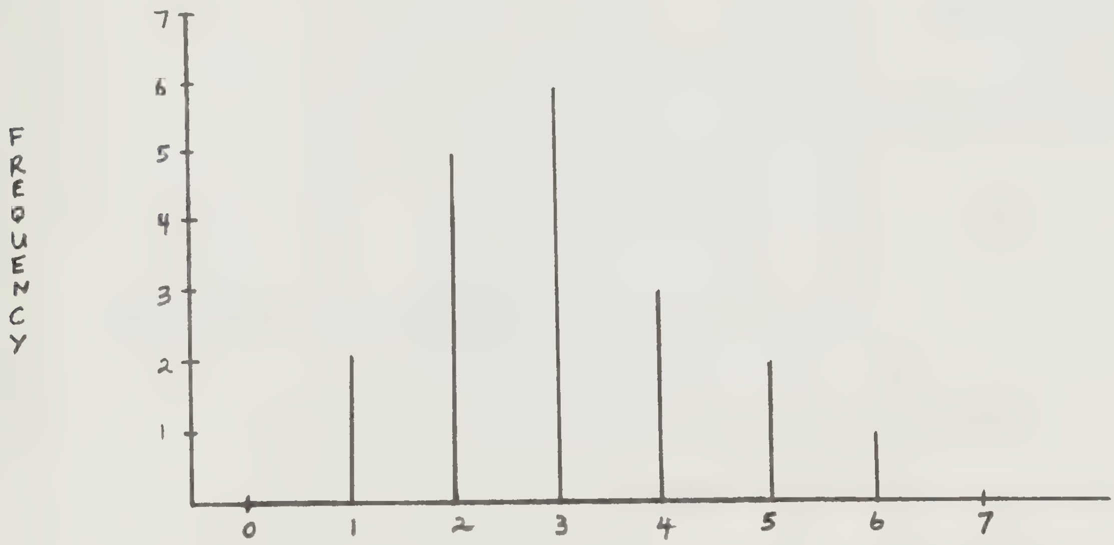
Populations of cities, type of eye colour, and popularity of hit songs are examples of data that can be suitably represented by bar graphs.

Example

The distribution of the number of pencils in each student's pencil case for class 9G is illustrated below.

Number of pencils	0	1	2	3	4	5	6
Frequency	0	2	5	6	3	2	1

This chart shows that six students each had three pencils in their pencil cases. This data is discontinuous; there is either a pencil in the case, or there is not. This data can be represented as follows:



number of pencils in a pencil case

Histograms

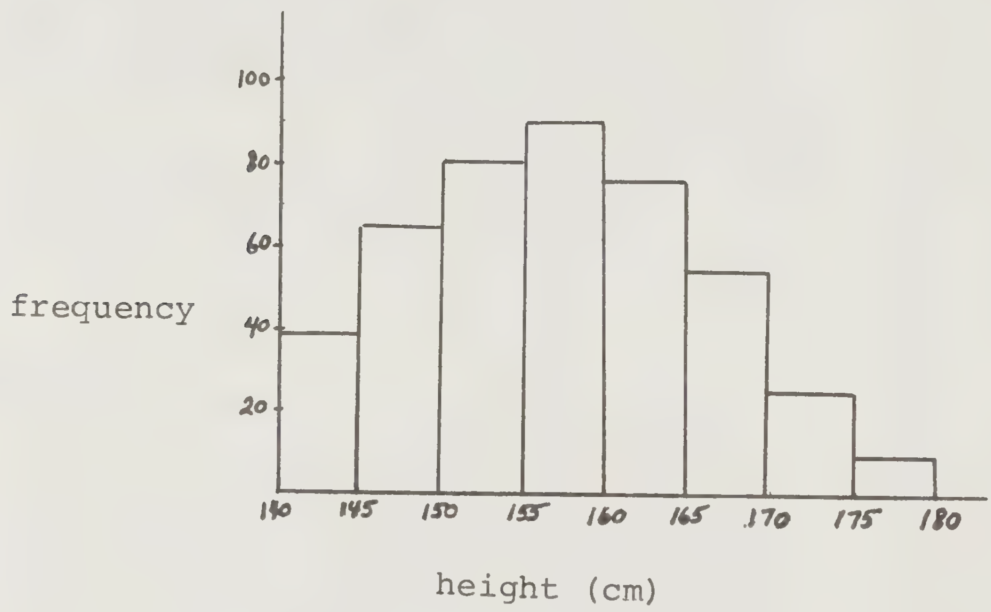
The histogram represents data that is related in some functional sense. Although the range is related to the domain, it is not always possible to express the relationship by an equation.

The histogram looks similar to a bar graph: the bars are vertical but without spaces in between.

Example

The height distribution of students may be represented by a histogram, since height is a continuous quantity: there are no sudden gaps in the domain. See the histogram below.

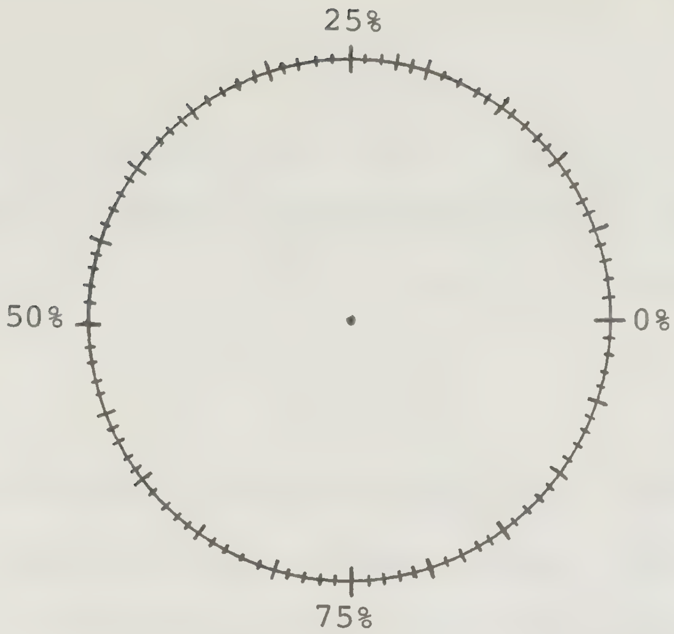
Height Distribution of Students in ACADEMY HIGH, September 1978



Circle Graphs

Grade 9 students are usually familiar with circle graphs. They may be reviewed here and some practice given on changing fractions to decimals, decimals to percentages, and the percentages to sizes of angles in the circle graph.

Some teachers may wish to provide the students with circles marked in percentage form, as illustrated below. The teacher may prepare them with dittos.



Pictographs and other methods

A major consideration in representing data is the quality of the visual impact of the graph. Does the graph have a strong visual appeal? Can the information be readily understood? Students who have some artistic ability may wish to present the data in colourful and original ways. The class should be encouraged to collect examples of such graphical representations. These may be used to catch the imagination of students and to encourage them to try something similar in their own work. For example:

Populations of Canada's Major Metropolitan Cities, 1976

d) Sample types

Students in this program need only consider random and stratified samples.

Random Samples

This topic can be introduced by involving the students in forming a small random sample for discussion purposes. The following examples illustrate ways of doing this with an overhead projector.

Example 1

Display the following grid.

1	6	0	10
2	3	8	11
5	7	13	15
9	12	14	4

Ask each student to select two (or more) numbers from the grid and write them down. The data collected from this simple process, when analysed, will often indicate that numbers near the centre are more likely to be chosen than numbers along the edges. This phenomenon has been observed in some European lotteries in which people pick their numbers from a rectangular array. Superstitions associated with the numbers 7 and 13 might also be considered.

Example 2

Project the following list with an overhead projector.

Jane Small

Jill Strong

Jean Smith

Joy Smythe

Ask the students to select one name and write it down.

There is usually evidence that certain positions in a list are preferred above others. This phenomenon has been shown to be statistically significant in the results of ballots cast in the British general elections. Connotations associated with words such as strong and joy as compared to small might be investigated.

Example 3

Ask the students to draw a profile (side view) of a face. How many profiles face left and how many face right?

The purpose of the above examples is to illustrate that many selection processes, which on the surface may appear to be random, in fact, turn out to be biased. Examples such as the above should help students to be critical of conditions that are associated with the supposedly random-sampling processes. In an ideal random sample, every event should have the same chance of occurring.

Students should discuss situations in which truly random selections will occur. Some of these are:

. flipping the pages of a book, stopping suddenly and selecting the last digit of the page number;

. using a set of commercially prepared digits;

. rolling a single die;

. turning a roulette wheel;

. using the second hand of a wrist watch or clock and, on a precise command, writing down the last digit of the seconds readout. (This is the starting point in one computer program for generating random numbers.)

Stratified Samples

This topic may be introduced by a discussion of the concept of proportional representation as advocated by one political party. This is illustrated in Example 1.

Example 1

Mazeria has a population of 10 million people. There are three political parties: the Consumeratives; the Liberationists; and the Old Anarchist Party. On a popular vote the population is split as follows: 5 million Consumeratives

3 million Liberationists

2 million O.A.P.'s

If there are 180 seats in parliament filled on the basis of popular vote, how are they likely to be split up among the various parties? (Of course this is a hypothetical situation designed to illustrate how the seats would be assigned if the assignment were proportional to the popular vote. In countries such as Canada, the country is divided into regions and the seats are filled on the basis of popular

vote for each region. Teachers might discuss this point and give examples of occasions where the party that won the national popular vote did not form the government.)

Answer:

Number of Consumerative seats = $\frac{5}{10} \times 180 = 90$

Number of Liberationist seats = $\frac{3}{10} \times 180 = 54$

Number of O.A.P. seats = $\frac{2}{10} \times 180 = 36$

Example 2

In a certain class there are 20 boys and 10 girls. We want to sample six students in order to estimate the average height of the class.

How many boys and how many girls should be in the sample? Why would you not use the same number of boys and girls?

e) Practical experience in data surveying

Students should experience the difficulties and frustrations of organizing and carrying out a data survey involving a fairly large sample. The survey should be aimed at organizing information that has social, economic, or other significance for the students. One such survey should be sufficient, since these surveys are time consuming and can be bothersome to the local community.

f) Determining the mean, median, and mode

This topic should be developed in conjunction with the next topic which deals with the appropriateness of the measures. There is little purpose in knowing how to determine the mean, median, and mode if the student does not know which of these measures is appropriate for a given situation.

The routine procedures for determining each of these measures are included in many intermediate level textbooks, and so need not be discussed here.

It is worth noting when finding the mean of data in which the same value occurs many times that the mechanical work can be reduced by grouping the same values into bundles. See the example below.

Example

Find the mean of the following values:

3, 5, 6, 3, 4, 5, 2, 4, 3, 3, 5, 6, 3, 4, 2, 6, 5, 4, 5, 3, 2, 6

The sum of the values may be written as:

$$\begin{aligned} & 2 \times 3 + 3 \times 6 + 4 \times 4 + 5 \times 5 + 6 \times 4 \\ = \; & 6 + 18 + 16 + 25 + 24 \\ = \; & 89 \end{aligned}$$

Therefore, the mean $= \frac{89}{22} \cong 4.05$

Students will appreciate that this method is implicit in a frequency table. Refer to the second example in the notes for topic b) on page 3 of this section. The mean age of the students can be found by extending the frequency table as follows:

Age	Tally	Freq.	Age x Freq.
14		3	14 x 3 = 42
15		8	15 x 8 = 120
16		6	16 x 6 = 96
17		3	17 x 3 = 51
18		1	18 x 1 = 18
Totals		21	327

$$\text{Mean} = \frac{327}{21} \cong 15.57$$

g) Appropriateness of the mean, median, and mode as measures of central tendency

A number of numerical investigations should be carried out in order to show how the mean, median, and mode are affected by the nature of the data values. This will help to prepare the students for making decisions regarding which average is most meaningful in a given situation. The example below illustrates a simple numerical investigation.

Example

Given as data the following set of numbers:

2, 6, 7, 7, 7, 8, 9, 9, 26

i) Calculate the mean, median, and mode.

ii) Change the extreme value of 26 to some other value, say 12. Calculate the central measures again. Which measure changes the most?

iii) Change one of the central values; for example, change 7 to 9 (restoring 12 to 26). Calculate the central measures again. Which measures changed significantly this time?

The appropriate use of each measure of central tendency may be developed by:

- examining a number of situations for which an average is needed;
- determining the mean, median, and mode for each situation;
- comparing the results and discussing their significance.

After a variety of these practical experiences, the students should be asked to examine a number of new situations to determine which average would be most significant and then to determine this average.

Eventually the students should establish opinions such as:

- the mode is not useful unless the modal frequency is much higher than any other frequency;
- the mean is affected by all of the values, and should be used when all the data values are significant to the final result (for example, the mass of each person in a tug-of-war team is equally important; thus the mean value is the most significant average for the mass of the team);

- the median and the mode are not affected by the extreme data values (for example, the median would be the most suitable measure for the average height of the student body if by chance the sample included the basketball team).

The following examples may clarify the above ideas.

Examples

1) When a doctor says that, on the average, influenza lasts about five days, he is using the median. Some people get over it in a couple of days, and some unfortunate people may take a very long time; but, by and large, five days is the average.
2) When an anthropologist studies the characteristics of a group, the average characteristic is the median. For example, the average height of central African pygmies is about 1.25 m. Slightly fewer than half of the pygmies are taller, and slightly fewer than half are shorter; only a relatively small number will be 1.25 m tall.
3) Most people are born with ten fingers; a very small number of people are born with fewer or more. Suppose for the world population that the mean value of the number of fingers is 10.000 000 000 1. Obviously this use of mean is inappropriate. The mode should be used in this case.

Ministry of Education Ontario

GRADE 9 GENERAL NUMERICAL METHODS

SECTION 3: POWERS, ROOTS

RELATED SECTIONS AND TOPICS

PAST	FY:	Pages 6, 11
	Ed PJ Div:	Pages 68-71
	Gr 7:	N 2; N 5; N 8; G 5bc
	Gr 8:	N 1; N 5; N 6; N 7; A 1bcd; A 4ad; G 3a; G 4bc
PRESENT	Gr 9 Gen:	N 1; N 4; N 6b-e; A 1; A 2b; G 2e; G 4ac; G 5bc
	Gr 9 Adv:	N 3; N 4; N 5bcdef; A 1; A 2c; G 2c-f
FUTURE	Gr 10 Gen:	N 1; N 4; N 7; G 1b; G 2c; G 3
	Gr 10 Adv:	N 1; N 2; N 3; G 1ce; G 2b; G 5

a) Decimal approximations of square roots; applications in real life situations

Decimal approximations

The concept of square root may be re-established by reviewing the table of squares from 1 to 12 from both the square and square root aspects. The table can then be extended to 20, and possibly beyond. It will be advantageous for students to learn by memory the squares of the numbers from 1 to 20. The idea that the radical sign $\sqrt{}$ and the exponent 2 indicate inverse operations should be established.

n	n^2	n	n^2
1	1	11	121
2	4	12	144
3	9	13	169
4	16	14	196
5	25	15	225
6	36	16	256
7	49	17	289
8	64	18	324
9	81	19	361
10	100	20	400

It should be mentioned that each positive number has two square roots – one positive and one negative. The existence of the negative square root is largely academic at this stage. Negative square roots become important when solving equations of degree 2 or higher. Further, it should be mentioned that negative numbers do not have square roots (in the real number domain).

By observing from the table that $\sqrt{1} = 1$, and $\sqrt{4} = 2$, the students should conclude that $\sqrt{2}$ and $\sqrt{3}$ cannot be integers, and must lie between 1 and 2.

At this point, an estimation approach is probably the best way of calculating square roots of numbers that are not perfect squares. In estimating the non-integral roots, the students may at first tend to believe that $\sqrt{2}$ should lie half way between 1 and 2 (that is, at 1.5). However, if they deal with both $\sqrt{2}$ and $\sqrt{3}$ simultaneously they will likely place the value of $\sqrt{2}$ below 1.5 and of $\sqrt{3}$ above. Typical estimates might be 1.4 and 1.6. At this stage a quick check by squaring will produce a new estimate for $\sqrt{3}$ larger than 1.6 – possibly at 1.7.

With the aid of a calculator, a student can now estimate (to one decimal place) the non-integral square roots of numbers up to 100. This can be done by a systematic 'guess and test' method using the concept of lower and upper bounds. Using a calculator that does not have a square root key will help to reinforce the concept of square root. Otherwise the student may be disinterested in doing a lengthy task when there is a quicker method available.

Applications

An interesting application involves calculating the time that it takes an eraser to fall from a student's desk to the floor. Using the formula $t = 0.45\sqrt{d}$, where d is the distance the eraser falls in metres and t is the time in seconds. For example, if the desk is 73 cm high, then $t = 0.45\sqrt{73} \cong 0.38$. Now the students may investigate similar situations for other heights. This application illustrates the process of selecting an algebraic model that represents the time-aspect of the real-world situation. In this case, the teacher should provide the model for the students, asking them to calculate the time for various measured heights, and then to check by clocking the times using a stop watch.

The history of the formula might be discussed. Students should be asked to discuss the accuracy of the two methods, formula and stop-watch. This could lead to a discussion of the need for reliable mathematical models, the time saved by using them, and the accuracy of the information they provide.

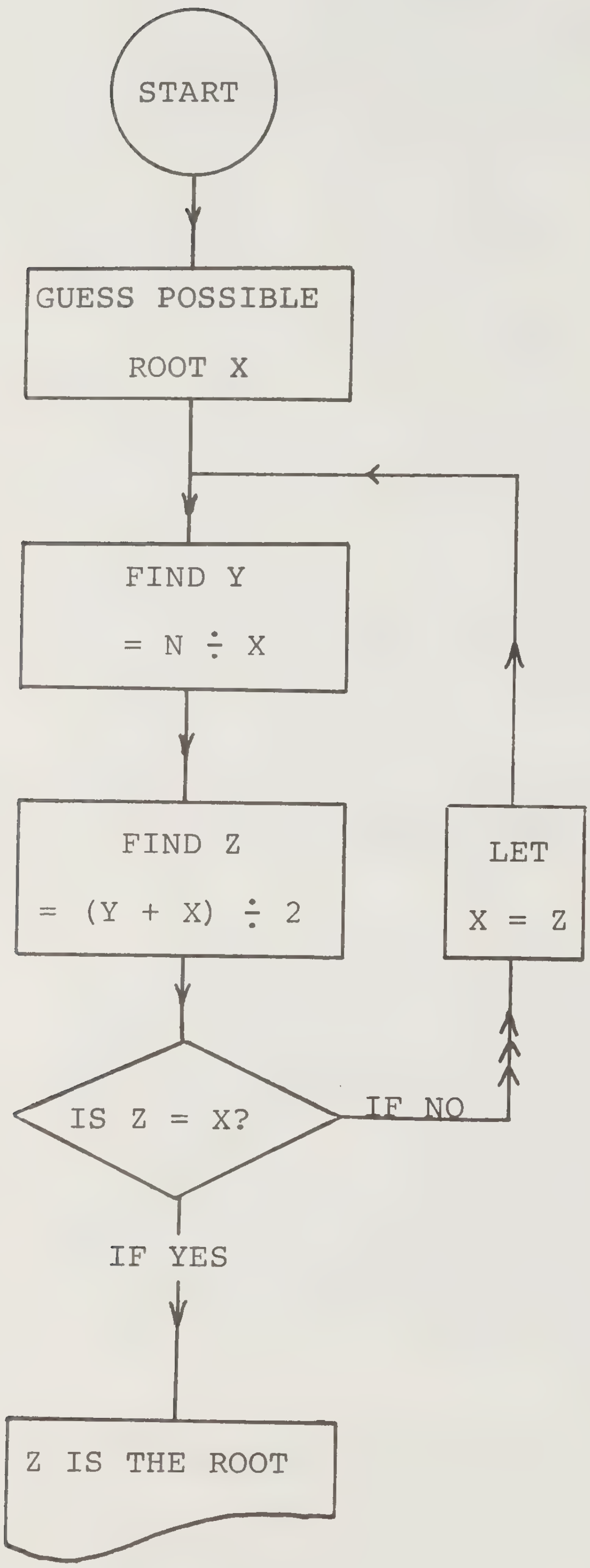
Another application involves a comparison of the distance travelled by a rook (castle) and by a bishop in a game of chess. A rook can travel only in a direction that is parallel to a side of the chess board. A bishop can travel only diagonally. Consider the side of a square to be one unit long. Which piece travels farther and by how much? Complete the following table and pattern to find out.

Number of squares travelled	Rook's horizontal distance in units	Bishop's diagonal distance in units
1	1	$\sqrt{1^2 + 1^2} =$
2	2	$\sqrt{2^2 + 2^2} =$
3	3	$\sqrt{3^2 + 3^2} =$
4	4	
5	5	
6	6	
7	7	

b) Newton's method for calculating square roots

Newton's method for finding a square root involves an iterative technique that can be effectively presented by a flow chart containing a loop, as illustrated below. The calculations can be made with a calculator. If the calculator is equipped with a memory, then each new value for X may be entered in memory and the calculation of Z made directly as Z = (N ÷ M) + M) ÷ 2.

Finding the Square Root of a Number N



N = 15

X = 4 (in memory)

* Z = ((15 ÷ M) + M) ÷ 2

= 3.875

= X (in memory)

Z = ((15 ÷ M) + M) ÷ 2

= 3.8729838

= X (in memory)

Z = ((15 ÷ M) + M) ÷ 2

= 3.8729833

= X (in memory)

Z = ((15 ÷ M) + M) ÷ 2

= 3.8729833

$\therefore \sqrt{15} \cong 3.8729833$

---

* M is the value in memory

c) Using the Pythagorean Theorem to solve problems in practical situations

Pythagorean Theorem

The Pythagorean Theorem is introduced in Grade 8 G 3a). It is suggested that the teacher gear the treatment for this topic to the students' present knowledge. The notes for teachers for 8G 3a) indicate ways by which this topic may be introduced and reinforced. Another way of investigating this theorem is shown below; others abound in the literature.

i) Prepare a figure on poster board as shown in figure 1. It consists of two right-angled triangles 1,2, and pentagon 3; the combined figure forms two squares, a^2 and b^2. Cut out figures 1,2, and 3 to form tiles.

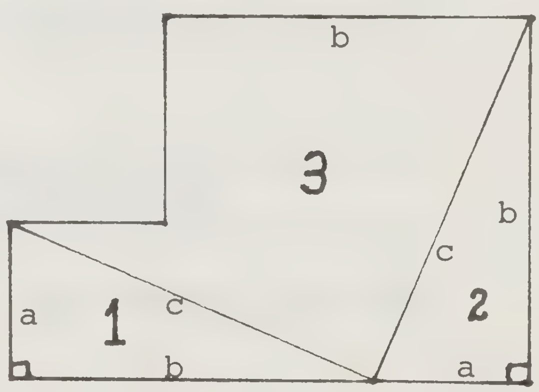


figure 1

ii) Re-assemble the tiles on an overhead projector to fit in a dashed outline as shown in figure 2.

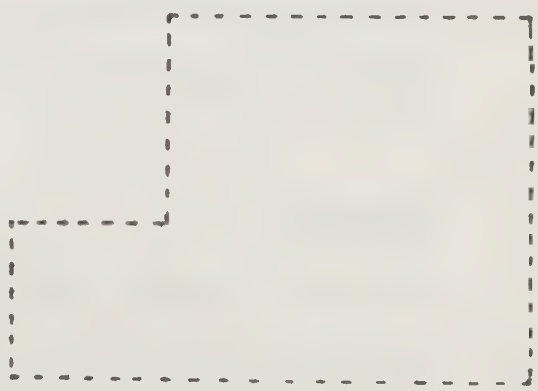
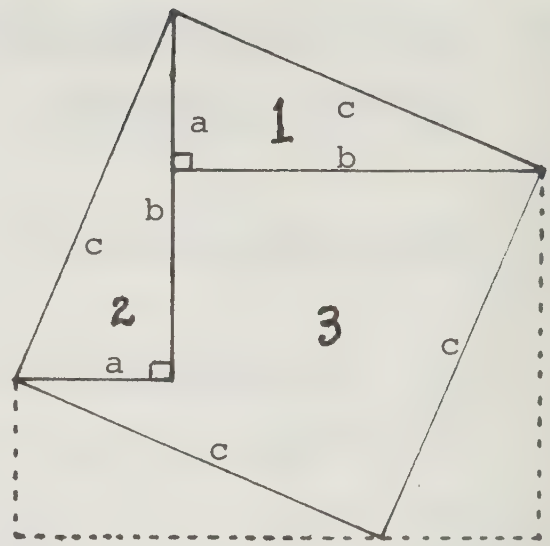


figure 2

iii) Slide triangle 1, then triangle 2, so that they fit as shown in figure 3. The figure so formed is a square with area c^2.



It follows that $c^2 = a^2 + b^2$.

Students should be encouraged to find out whether the Pythagorean relationship holds for triangles that are not right-angled.

<u>Problems in practical situations</u>

At this point the students should investigate problems (within their own experiences) that involve the Pythagorean Theorem.

<u>Example 1</u>

Determine the length of the longest line that can be drawn on this page, in order to decide how long a ruler will be necessary in mathematics class.

<u>Example 2</u>

How long is the path that cuts diagonally across a corner lot that is 18 m x 36 m?

d) Powers with integral exponents, their evaluation

The concept of power notation with positive integers as exponents can be effectively reinforced and extended to negative exponents through the use of the calculator. The calculator enables students to evaluate powers with relatively large exponents both quickly and accurately, and to explore patterns that suggest the meaning of negative integral exponents. The fact that a power is merely an abbreviated symbol for successive multiplications of the same factor can be emphasized by asking students to evaluate powers such as the following.

Example 1

Evaluate 3^6.

Key diagram	Display	Power calculated
	0.	
$\boxed{3}$	3.	3^1
$\boxed{x}$	3.	
$\boxed{=}$	9.	3^2
$\boxed{=}$	27.	3^3
$\boxed{=}$	81.	3^4
$\boxed{=}$	243.	3^5
$\boxed{=}$	729.	3^6

Example 2

Evaluate $(-5)^8$.

Key diagram	Display	Power calculated
	0.	
$\boxed{5}$ $\boxed{+/-}$ or $\boxed{-}$	0.	
$\boxed{5}$	-5.	$(-5)^1$
$\boxed{x}$	-5.	
$\boxed{=}$	25.	$(-5)^2$
$\boxed{=}$	-125.	$(-5)^3$
$\boxed{=}$	625.	$(-5)^4$
$\boxed{=}$	-3125.	$(-5)^5$
$\boxed{=}$	15625.	$(-5)^6$
$\boxed{=}$	-78125.	$(-5)^7$
$\boxed{=}$	390625.	$(-5)^8$

It should be noted that:

- when the base -5 is entered, the degree of the power is one;
- each successive entry of the equal sign increases the exponent by one;
- the product is alternately negative and positive: an odd exponent produces a negative answer, an even exponent a positive result.

At this stage the students may investigate the inverses of the operations in the previous examples. This will lead naturally to the discovery of the meaning of negative integral exponents. For example, since evaluating 3^6 involves successive multiplications, the inverse operations must involve successive divisions. After completing and recording Example 1, students could be given the following table and asked to extend it by entering a succession of 11 equal signs.

Key Diagram	Display	Power Calculated
	0.	
$\boxed{7}$	7.	-
$\boxed{2}$	72.	-
$\boxed{9}$	729.	3^6
$\boxed{\div}$	729.	-
$\boxed{3}$	3.	-
$\boxed{=}$	243.	3^5

In carrying out this investigation, the students should observe that each time the $\boxed{=}$ sign is entered:

- the display is one-third the previous display;
- the power they write is one-third the previous power (for example, $3^5 = \frac{1}{3} \times 3^6$), and its exponent is one less than the previous exponent — $3^6, 3^5, 3^4, 3^3, \ldots$

On the basis of the above observations, the students should be able to extend the list of powers to show that:

- 3^0 has a value of 1;
- 3^{-1} has a value of 0.3333333 and is equivalent to $\frac{1}{3}$;
- 3^{-2} has a value of 0.1111111 and is equivalent to $\frac{1}{9}$ or $\frac{1}{3^2}$;
- and so on.

This is illustrated in the first three columns of the following chart. The fourth column may be added later; it reinforces the relation between 3^{-6} and 3^6, 3^{-5} and 3^5 and so on.

Key Diagram	Display	Power Calculated	1 ÷ Display
	0.		
$\boxed{7}$	7.	-	
$\boxed{2}$	72.	-	
$\boxed{9}$	729.	3^6	0.0013717
$\boxed{\div}$	729.		
$\boxed{3}$	3.		
$\boxed{=}$	243.	3^5	0.0041152
$\boxed{=}$	81.	3^4	0.0123456
$\boxed{=}$	27.	3^3	0.037037
$\boxed{=}$	9.	3^2	0.1111111
$\boxed{=}$	3.	3^1	0.3333333
$\boxed{=}$	1.	3^0	1.
$\boxed{=}$	0.3333333	3^{-1}	3.0000003
$\boxed{=}$	0.1111111	3^{-2}	9.0000009
$\boxed{=}$	0.037037	3^{-3}	27.000027
$\boxed{=}$	0.0123456	3^{-4}	81.000518
$\boxed{=}$	0.0041152	3^{-5}	243.00155
$\boxed{=}$	0.0013717	3^{-6}	729.02238

The discrepancies that occur after the decimal point in the last six entries of the fourth column should be discussed. A discussion of errors resulting from rounding and truncating can be found in Calculators and Numerical Methods, a resource module for Intermediate Division Mathematics.

The student should note the following:

POWER	INVERSE
$3^6 = 729$	$\frac{1}{3^6} = \frac{1}{729} = 0.0013717 = 3^{-6}$
$3^{-6} = 0.0013717$	$\frac{1}{3^{-6}} = \frac{1}{0.0013717} = 729 = 3^6$

A study of this small table should lead to the observation that $3^{-6} = \frac{1}{3^6}$ and $3^6 = \frac{1}{3^{-6}}$. An investigation of other powers of 3 should lead to the establishment of the general definition for a power with a negative integer as exponent; $b^{-n} = \frac{1}{b^n}$ where n is a whole number.

e) Laws for multiplication and division with powers developed from numerical cases

Reference to a table such as the one above for powers of 3 along with the use of a calculator will assist the students in developing the laws for multiplication and division with powers.

	$3^2 \times 3^4$	$= 9 \times 81$	(from the table)
		$= 729$	(pencil and paper, or calcula
		$= 3^6$	(from the table)
Similarly	$3^2 \times 3^{-4}$	$= 9 \times 0.0123456$	(from the table)
		$= 0.1111104$	(by calculator)
		$= 3^{-2}$	(from the table)

By investigating other examples similar to these, the students should be able to establish the law for multiplying powers with the same base and integral exponents.

The investigation should also include multiplication involving powers with different bases. Students could make use of any tables of powers they have developed in topic d) to see whether the law for multiplication still holds. For instance, they might refer to tables for powers of 2 and 3. They yield the following solution:

$$2^4 \times 3^2 = 16 \times 9$$
$$= 144$$

A search of the display column of the tables for powers of 2 and 3 will not locate 144, nor will the table for powers of 6. In particular the student might check the value of powers of each of these bases up to exponent 6.

The above exercise emphasizes the necessity that the bases be the same before the law for multiplication with powers can be applied.

The students could be encouraged to conjecture the law for division with powers having integral exponents before they actually begin to investigate it. To develop the law itself, the same tables and a method similar to that described for multiplication with powers could be employed. For example:

$$3^4 \div 3^{-2} = 81 \div 0.1111111 \quad \text{(from the table)}$$
$$= 729.00007 \quad \text{(by calculator)}$$
$$\cong 3^6 \quad \text{(from the table)}$$

Referring to the exponents

$$4 - (-2) = 4 + 2$$
$$= 6$$

After investigating a number of similar examples, the students should be able to confirm the law for division.

Again an investigation of division of powers with different bases and a search through available tables to attempt to express the answers in power notation should reinforce the fact that the law for division also applies only when the bases are the same.

f) Scientific notation; applications in other disciplines

The need for scientific notation can be established by a few well-chosen examples. A student could dictate each of the following sentences to the class by reading them only once with the other students copying what they hear without asking questions.

1. One red blood cell contains 270 000 000 haemoglobin molecules.
2. A bee's wing has a mass of 0.000 000 05 kg.
3. The Great Spiral Nebual in Andromeda is about 19 000 000 000 000 000 000 km away.

This process will likely result in some discrepancies in what the students think they hear and what they write. Once corrections have been made, students should be asked to suggest ways to achieve greater accuracy in handling such numbers. If it is suggested that triples of digits from the decimal point be separated by commas, the new metric style involving half-spaces should be explained.

i.e. 270 000 000 instead of 270,000,000;

0.000 000 05 instead of .00000005; and

19 000 000 000 000 000 000 instead of 19,000,000,000,000,000,000.

Consider also how a newspaper would print, "Santa Claus must travel to about 2 billion households on Christmas Eve." This may lead to the establishing of the following table for large numbers:

10^3	thousand	10^{21}	sextillion
10^6	million	10^{24}	septillion
10^9	billion	10^{27}	octillion
10^{12}	trillion	10^{30}	nonillion
10^{15}	quadrillion	10^{33}	decillion
10^{18}	quintillion	10^{100}	googol*

Now 2 billion households could be written 2×10^9. Note that if typed, '2 000 000 000' takes 13 spaces (assuming half-spaces are not available), '2 billion' takes 9 spaces, but 2×10^9 takes only 7 spaces. Note also that the need for scientific notation has been identified and so provides motivation for its use.

A search for interesting bits of information will pay off in giving the student the necessary practice with an otherwise drill-oriented skill. Ask students to find statements of the following kind in which the numbers should be converted to scientific notation:

---

* Most of the prefixes can be related to other words that are within the student's vocabulary - <u>quint</u>uplets, <u>oct</u>ave, <u>dec</u>imal. "Googol" was a word that a young boy made up to represent a very large number. His uncle was the American mathematician, Edward Kasner.